I PRINCIPALI LIMITI NOTEVOLI

LIMITE DI NEPERO

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$$

DERIVATI DAL LIMITE DI NEPERO

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=1 \qquad \Rightarrow \qquad \ln(1+x)\underset{x\to 0}{\sim} x$$

$$\lim_{x\to 0}\frac{e^{x}-1}{x}=1 \qquad \Rightarrow \qquad e^{x}\underset{x\to 0}{\sim} 1+x$$

$$\lim_{x\to 0}\frac{(1+x)^{\alpha}-1}{x}=\alpha \qquad \Rightarrow \qquad (1+x)^{\alpha}\underset{x\to 0}{\sim} 1+\alpha x$$

LIMITI GONIOMETRICI

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \Rightarrow \qquad \sin x\underset{x\to 0}{\sim} x$$

$$\lim_{x\to 0}\frac{\tan x}{x}=1 \qquad \Rightarrow \qquad \tan x\underset{x\to 0}{\sim} x$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^{2}}=\frac{1}{2} \qquad \Rightarrow \qquad \cos x\underset{x\to 0}{\sim} 1-\frac{x^{2}}{2}$$

Utile:

$$\lim_{x\to 0}\ x^{\alpha}\ln x=0 \ \text{ se } \alpha>0$$

SVILUPPI DI MC LAURIN

$$e^x = \sum_{n=0}^{+\infty} \frac{1}{n!} x^n = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o\left(x^3\right)$$

$$\sin x = \sum_{n=0}^{+\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + o\left(x^5\right)$$

$$\cos x = \sum_{n=0}^{+\infty} (-1)^n \frac{1}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o\left(x^4\right)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + o(x^9)$$

$$\ln(1+x) = \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{1}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} + o\left(x^3\right)$$

$$(1+x)^\alpha = \sum_{n=0}^{+\infty} \binom{\alpha}{n} x^n = \sum_{n=0}^{+\infty} \frac{\alpha(\alpha-1)\cdot\ldots\cdot(\alpha-n+1)}{n!} x^n = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + o\left(x^3\right)$$

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + o\left(x^5\right)$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{122} + \frac{35x^9}{1152} + o\left(x^9\right)$$

$$\arccos x = \frac{\pi}{2} x - \frac{x^3}{6} - \frac{3x^5}{40} - \frac{5x^7}{122} - \frac{35x^9}{1152} + o\left(x^9\right)$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + o\left(x^5\right)$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + o\left(x^4\right)$$

$$\tanh^{-1} x = 1 + \frac{x^3}{3} + \frac{x^5}{5} + o\left(x^5\right)$$

$$\sinh^{-1} x = x - \frac{x^3}{6} + \frac{3x^5}{40} - \frac{5x^7}{122} + \frac{35x^9}{1152} + o\left(x^9\right)$$